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Cyclic and Secular Variations Sunspot Groups with Various Scales

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Abstract—Data from the Greenwich Catalog and its NOAA–USEF extension are used to analyze the spot-formation activity on the Sun separately for small ($S < 100$ msh), medium ($100 < S < 500$ msh), and large ($S > 500$ msh) sunspot groups. The relationship between the numbers of groups with various areas changes with time. This is determined primarily by numerous small-area groups. Over nearly 150 years, periods have been observed when the relative number of large groups has increased (Cycles 18 and 19), as well as extensive periods when the number of small groups has grown. As a rule, the latter correspond to low activity cycles. The observed relations indicate the possible interaction of two independent mechanisms in the spot-formation activity of the Sun. A deep dynamo controls the variations of the number of small spots, while the formation of large spots is determined by processes in sub-surface layers.

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1. INTRODUCTION

In the analysis of variations of the spot-formation activity of the Sun, attention has usually focused on the number of sunspots, more precisely the so-called Wolf number. This index was calculated in Zurich up to 1981; since 1981, the responsibility for calculating this index has been held by the Royal Observatory of Belgium in Uccle. The index acquired a new name—the international sunspot number (Wolf number)—and is usually denoted R_Z. In spite of numerous methodical changes in the procedure used to calculate this index, which hinder treating this as a completely uniform series [1–4], the main formula used to determine R_Z has not changed over many years:

$$R_Z = k(10G + N), \qquad (1)$$

where G is the total number of sunspot groups on a given day, N the total number of spots in all groups on the same day, and k a coefficient determined by the properties of the observatory and the quality of the observations. The monthly-mean daily value of R_Z is usually calculated. Smoothing of the monthly-mean sunspot number with a 13-month window is also often used.

Time variations in R_Z associated with variations in the number of sunspots do not take into account variations in the relative contributions of large and small sunspot groups. Until very recently, possible variations in the relationships between groups with various areas with the phase of the solar cycle were not considered. In their well known monograph on the statistics of spot-formation activity on the Sun, Vitinskii et al. [1] simply did not consider this question. This issue is given only one page in the monograph [5]. At the same time, there is no basis to suppose *a priori* that the relative contributions of sunspot groups with different sizes do not change within a cycle, as well as on longer time scales. Moreover, such variations have been observed in a number of studies. For example, this question was considered in [6, 7] in relation to active regions for data obtained in 1967–1981, in the decay phase of the 20th cycle and the growth phase of the 21st cycle. The fraction of developed groups (which, as a rule, also have larger areas) was found to be higher near the cycle maximum. It was shown in [8] that the incidence of short-lived groups depends strongly on the phase of the secular cycle. However, until recently, this question was not considered based on a collection of data obtained over many cycles.

A deficit of small spots on the descending branch of the 23rd cycle was found in [9]. At the same time, Nagovitsyn et al. [10] associate the fall in the maximum magnetic-field strength in sunspots detected in [11] with an increased number of small spots. Bludova et al. [12] drew a similar conclusion based on an analysis of variations of the relative fraction of sunspots occupied by the umbra. This result was recently confirmed in [13, 14]. Javaraiah [13] analyzed variations of the yearly number of small (areas at the maximum of their development $S < 100$ millionths of a solar hemisphere, msh), large

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($100 < S < 300$ msh), and very large ($S > 300$ msh) sunspot groups. As in [12], the Greenwich data series and its NOAA–USEF extension were used. The Gnevyshev–Ohl rule was confirmed for both large and small sunspot groups. This rule was violated for small spots in two cycles (22, 23), and for very large groups in two pairs of cycles (12 and 13, 22 and 23).

2. VARIATIONS IN THE RELATIVE FRACTIONS OF LARGE AND SMALL SPOTS AS A FUNCTION OF TIME AND PHASE OF THE CYCLE

Let us begin our analysis by comparing the cyclic variations of three sets of spot groups: small sunspot groups with areas $S < 100$ msh, large groups with areas $S > 500$ msh, and groups with intermediate areas $100 < S < 500$ msh. The calculations were carried out using the Greenwich Catalog and its NOAA–USEF extension (`http://solarscience.msfc.nasa.gov/greenwch.shtml`). Note that our class of small groups coincides with the class of small groups adopted in [13, 14], while our class of large groups is somewhat more strictly defined than the class of very large groups (more than 300 msh) adopted in [13, 14]. However, another difference is more important: we considered the daily areas of the groups, and not the area at the development maximum. Since, as a rule, larger groups live longer than small ones, our procedure leads to an increase in the relative fraction of large (and a corresponding decrease in the relative fraction of small) groups, compared to [13, 14]. We believe that this approach is mre suitable for studying the characteristics of spot-formation activity, since it explicitly takes into account the overall power of processes. Moreover, this approach makes it easier to compare with R_Z, since formula (1) contains the total number of groups observed on the solar disk on a given day, independent of whether or not one or another group is observed on other days.

We calculated the total number of groups observed over a month (the incidence) for each class, independent of how many days each group was observed. We denote the total number of small and large groups observed over a month G_S and G_L, and the total number of all groups G_T. The total monthly number for groups with medium-size areas $100 < S < 500$ msh is denoted G_M. The total monthly data were averaged using a sliding window with a duration of 13 months. These are the yearly smoothed data used below.

Figure 1 presents the cyclic variations of these sets of sunspot groups, together with the cyclic variations of the total number of sunspot groups. For both the large and small groups, the time dependences of the smoothed monthly numbers of sunspot groups do not coincide with the dependence for all spots. The highest cycle for large groups (Fig. 1, lower plot), is Cycle 19 (as is also true for the set of all sunspot groups and for the Wolf number). However, Cycles 20, 21, 22, and 23 all have the same height, which differs sharply from the height ratios for these cycles for the Wolf numbers. The differences observed for small spot groups are even more substantial (Fig. 1, second graph from the top). Here, Cycle 19 does not stand out, and the most prominent cycles are 21 and 22. In contrast to the assertion made in [9], there is no clear deficit of small spot groups on the descending branch of Cycle 23 in Fig. 1. However, the more detailed analysis carried out in [15] confirms the decrease in the number of small sunspot groups.

The similarities and differences of the cyclic variations in the numbers of groups of different sizes are manifest most clearly when these are compared at the cycle maxima. Figure 2 compares the numbers of groups of various sizes at the cycle maxima, and presents the correlation relationships between them. The upper left panel plots the peak incidence of small groups G_S ($S < 100$ msh) against the peak incidence for all groups, G_T. Analogous diagrams for G_M ($100 < S < 500$ msh, upper right panel), and G_L ($S > 500$ msh; lower left panel) are also shown. The lower right panel presents the correlation between G_L and G_S.

The peak incidence for small groups G_S is well correlated with the peak incidence for all groups G_T (upper left panel in Fig. 2; correlation coefficient 0.925). This is expected, since small groups comprise an absolute majority (about two-thirds) of the total number of groups. The correlation between the peaks of G_M and G_T is also high (upper right panel; correlation coefficient 0.818). However, the correlation between G_L and the total number of groups G_T is much lower (lower left panel; correlation coefficient 0.579). The peak incidences for the large and small groups are essentially uncorrelated (lower right panel; correlation coefficient 0.258). The anomalously large number of large groups in Cycle 19 and anomalously low number of large groups for Cycles 21, 22, and 23, as well as the not yet completed Cycle 24, can be noted. Thus, a secular maximum of the solar activity was attained in Cycle 19, after which a general drop in solar activity began.

We can also directly compare the peak incidences for various-size groups with the maximum values of the index R_Z (table). The correlation coefficient between the maximum values of G_T and R_Z is very high, 0.959. The correlations of G_S, G_M, and G_L with R_Z differ from their correlations with G_T.

This is apparently due to the fact that the definition of the sunspot-number index R_Z contains the total number of spots in groups, with their particular

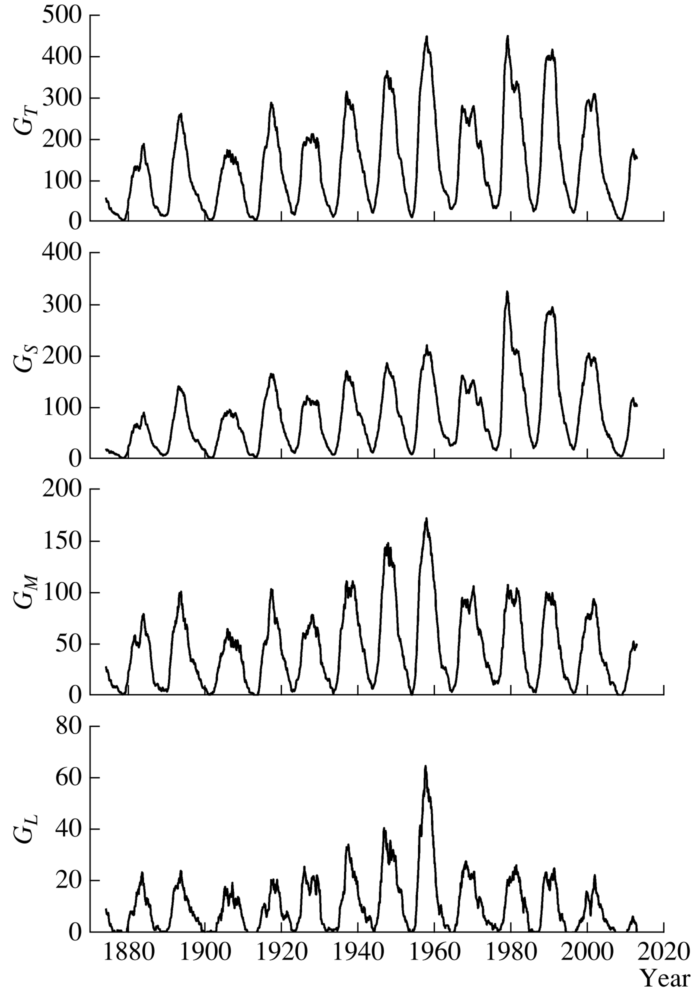

Fig. 1. Cyclic variations in the total number of sunspot groups and the numbers of groups with small, intermediate, and large areas (from top to bottom). The smoothed total monthly values are presented.

dependences on the areas of groups, their time variations, and the individual dependence on the area of the groups.

Correlations of the peak incidences of groups of sunspots of various sizes between each other and with R_Z

Parameter	Correlation coefficient				
	R_Z	G_T	G_S	G_M	G_L
R_Z	1	0.959	0.836	0.865	0.713
G_T	0.959	1	0.925	0.818	0.579
G_S	0.836	0.925	1		0.258
G_M	0.865	0.818		1	
G_L	0.713	0.579	0.258		1

The variation in the incidences of groups of various areas can be studied in more detail by constructing dependences of the relative numbers of sunspot groups α as a function of area, and fitting these dependences with exponential functions. We constructed these dependences within each cycle separately for the main characteristic intervals: minimum, growth of activity, maximum, and decay. The lengths of these characteristic intervals was two to three years. Our fitting was carried out using the formula

$$\alpha(S) = \alpha_0 + A_1 \exp[-(S - S_0)/t_1]. \tag{2}$$

Here, $\alpha(S)$ is the fraction of groups in a given range of areas normalized to the total number of groups G_T at a given phase of the cycle, and α_0, A_1, S_0, and t_1 are parameters that were found for each time interval through a least-squares fit. The area step used to construct the dependences was 100 msh. The time step was based on the characteristic intervals of the cycles, using the definition of the cycle phases

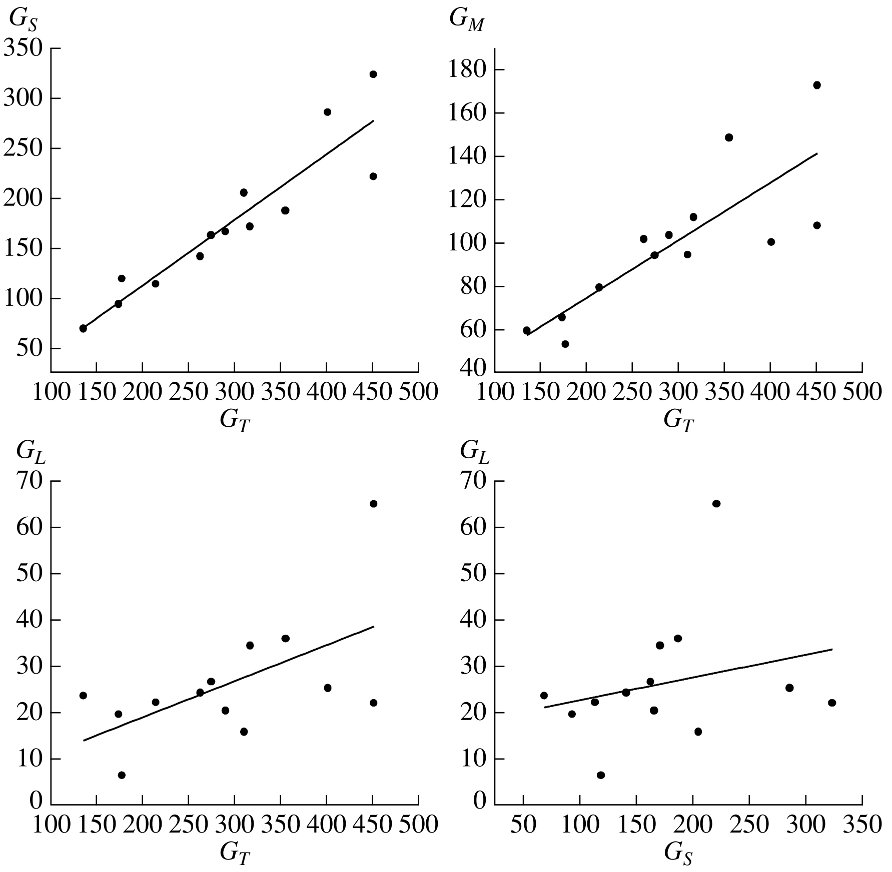

Fig. 2. Comparison of the numbers of groups of various sizes at cycle maxima.

from [16]:

$$\Phi = (\tau - m)/(|M - m|). \tag{3}$$

Here, τ is the current time and M and m the times of the nearest maximum and minimum of the 11-year cycle, respectively. According to (3), the phase is zero at the minimum of each solar-activity cycle, -1 at the maximum of the preceding cycle, and $+1$ at the maximum of the following cycle. The phase is positive on the growing branch of a cycle and negative on the descending branch. This definition of the phase incorporates normalization to the lengths of the growth and decay branches in each cycle.

We introduced the following definitions for the phases of the characteristic intervals of the cycles:
0—cycle minimum (from $\Phi = -0.3$ of the preceding cycle to $+0.3$ of the current cycle);
0.25—growth branch (from $+0.3$ to $+0.8$);
0.50—maximum (from $+0.8$ to -0.8);
0.75—decay branch (from -0.8 to -0.3).

An example of a dependence of α on S, for the decay branch of Cycle 16 (July 1929–February 1932), is shown in Fig. 3.

It is obvious that, the larger the value of t_1 in the fit (2), the longer the "tail" of the exponential dependence, which parametrizes the contribution of large groups. Thus, t_1 characterizes the relative contribution of groups with larger areas; the relative contribution of small groups grows as t_1 decreases. The time dependence of t_1 is shown in Fig. 4. The first point in Fig. 4 corresponds to the miminum between Cycles 11 and 12, and the last point to the growth interval of Cycle 24.

Figure 4 shows that the relative contrbution of large groups varies strongly with time, with suggestions of a secular cycle with a period of about 80 years. The contribution of the largest groups grows in the period of the high 18th and 19th cycles (between 1945 and 1960), and also at the beginning of the studied time interval. Relatively small values of t_1 are observed during relatively low cycles, during 1900–1920 and 1985–2000. Overall, the entire 20th century is characterized by a reduced fraction of large sunspot groups, and an enhanced fraction of small groups. At the end of the studied period, beginning with the 20th cycle, we again observe a gradual decrease in t_1, indicating an increase in the fraction of small sunspot groups. This is consistent with the conclusions of [10, 13–15], but contradicts the conclusions of [9].

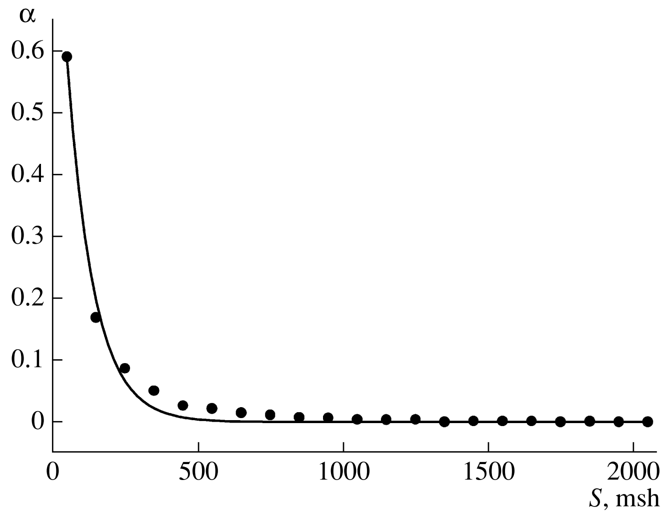

Fig. 3. Dependence of α on S for the decay of Cycle 16. The solid curve shows a fit using the exponential function (2).

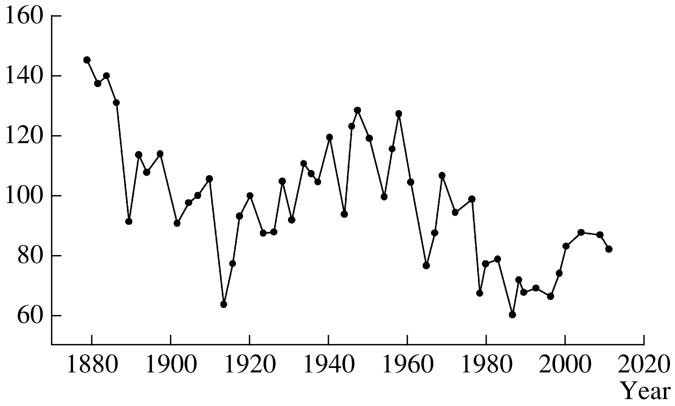

Fig. 4. Time variations of the parameter t_1 in (2).

A direct count of the number of small sunspot groups during 1874–2012 (Fig. 5) performed using the same input material shows a similar time behavior for the fraction of small sunspot groups, and some decrease in this fraction beginning from the descending branch of the 23rd cycle [15]. Here, the thin curve shows the time variations of the yearly fraction of small groups, while the bold curve shows a running average performed using a 25-year window. A certain decrease in the number of small sunspot groups can also be seen in Fig. 4.

In [12], we pointed out that two types of small sunspot groups can be distinguished: those containing ordinary spots possessing umbrae, and those containing unusual spots without umbrae. The temporal behavior of groups of sunspots without umbrae observed during 1874–1976 was considered in [17], where it was shown the the relative number of these objects varies with time; for example, virtually no such sunspots were observed in the 1930s.

Let us now consider separately the time behavior of the numbers of spots with and without umbrae. The relative fractions of groups of these two types of spot vary differently with time (Fig. 6). The number of small sunspot groups with areas up to 100 msh and possessing umbrae relative to the total number of sunspot groups α_1, has a maximum in the 1930s (upper curve in Fig. 6), when the corresponding relative number of sunspot groups without umbrae α_2 has a minimum (lower curve in Fig. 6). The bold curves show a running average performed using a 25-year window. These curves characterize the secular variations of α_1 and α_2.

Overall, Fig. 6 shows that the two dependences display anti-correlated behavior during 1874–1976, with increasing α_1 corresponding to decreasing α_2 and vice versa. The correlation coefficients for the

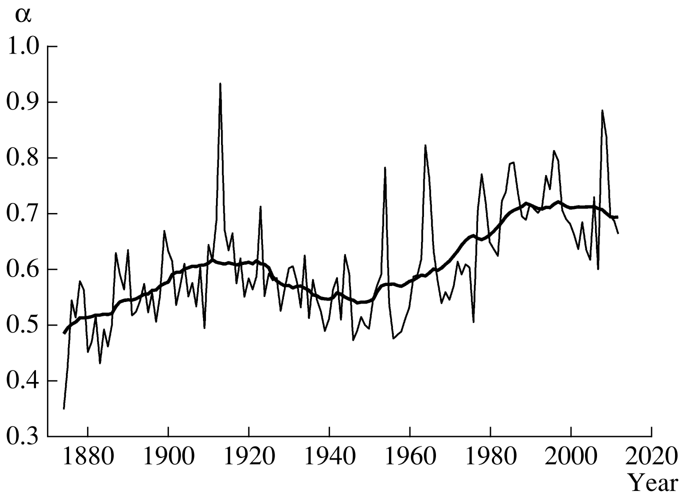

Fig. 5. Fraction of sunspot groups with areas up to 100 msh.

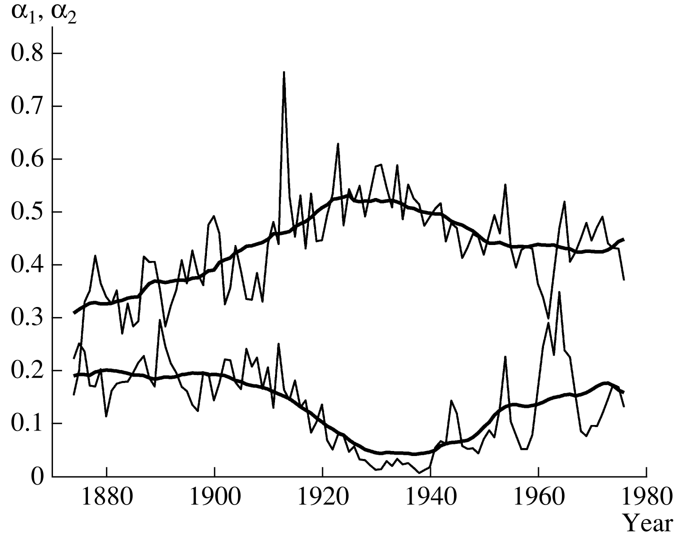

Fig. 6. Comparison of the fraction of small sunspot groups with umbrae α_1 and the fraction of sunspot groups without umbrae α_2. The bold curves show a running average with a 25-year window.

unsmoothed and smoothed data are −0.59 and −0.88, respectively. Unfortunately, the extension of the Greenwich catalog does not contain data on the sizes of the umbrae of spots in groups after 1976, making such an analysis impossible for subsequent years.

This behavior of α_1 and α_2 could have two explanations. On the one hand, it could be associated with uncertainty in the sizes of the total umbrae in sunspot groups; in other words, the observed anti-correlation between α_1 and α_2 could be an artefact.

On the other hand, a real decrease in α_2 (the relative fraction of sunspot groups without umbrae) accompanied by an increase in α_1 (the relative fraction of sunspot groups with umbrae) toward the beginning of the 1930s is possible. This is suggested by the observation of certain properties in the behavior of the solar activity at that time:

(a) the ratio of the total umbra area in a sunspot group to the total area of the group reached a maximum [12];
(b) no spots with complex magnetic configurations were observed [18];
(c) an increase in the rotational velocity of the Sun as a star was observed [19, 20].

Figure 7 (upper) presents a two-dimensional diagram showing the time variations of the relative fractions of sunspot groups of various sizes, $\alpha(S,t)$. The relative fraction of small sunspot groups increased sharply near 1917 and 1990, with this effect extending to some groups with intermediate areas. The relative number of large sunspot groups decreases in the

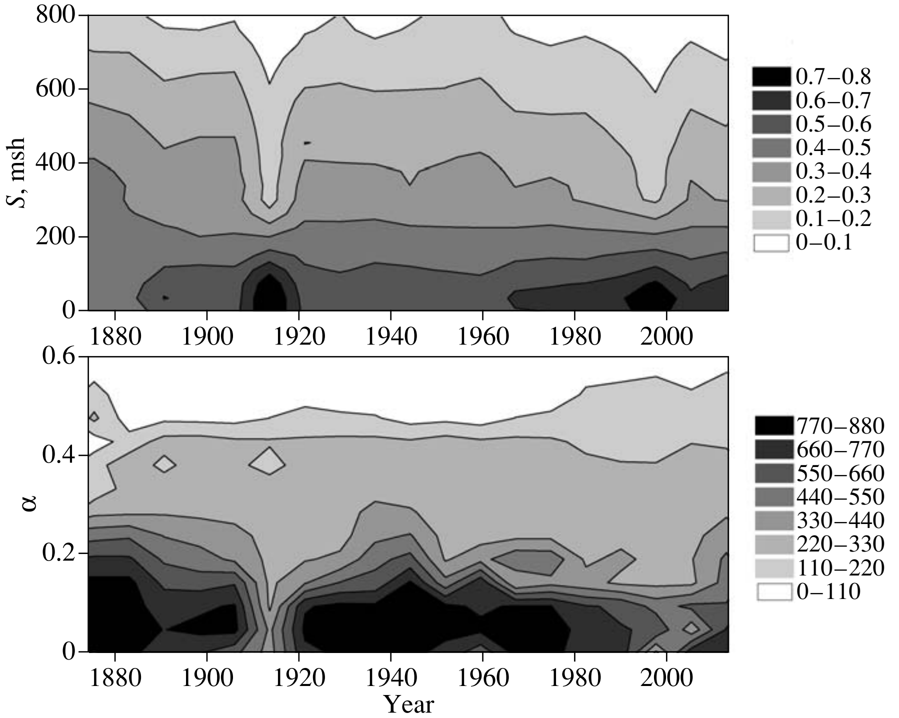

Fig. 7. Upper: variations in the relative fraction of sunspot groups of various sizes as a function of area, $\alpha(S, t)$. Lower: dependence of the area of sunspot groups on their relative incidence, $S(\alpha, t)$.

same periods, and this behavior also extends to some groups with intermediate areas. The relative fraction of sunspot groups with areas of about 300 msh remains essentially constant in time.

This same figure can be constructed in another way in order to estimate how the areas of groups change with time as a function of their incidence (Fig. 7, lower). The lower diagram in Fig. 7 $S(\alpha, t)$ again shows that very small groups with areas of less than 100 msh are most common. The most rare groups are those with areas of 770–880 msh. These form a stripe in the diagram corresponding to an incidence of less than 0.1. In special periods near 1917 and 1990, there are no such groups, and their place in terms of their low incidence is replaced by groups with areas of 330–550 msh, whose incidence is appreciably lower. The incidence of groups with areas of 100–300 msh remains essentially the same, changing from 0.2 to 0.4.

3. CYCLIC VARIATIONS OF GROUPS OF VARIOUS AREAS

Figurea 3–7 clearly show the presence of two periods when the solar surface displayed relatively many large sunspot groups during the interval from the start of the Greenwich series (1874) to the current date. The first corresponds to the beginning of this series (about 1880), and the second to the middle of the 20th century (1940–1960). Overall, these periods correspond approximately to the high Cycles 11 and 18–19. Two periods when the fraction of small groups also rose were also observed. The first was in 1910–1930, and corresponds to the relatively modest Cyles 14 and 15. The second period of increased incidence of small groups began in 1980 and has continued essentially to the current time. Both fairly high cycles (21 and 22) and modest cycles (23 and 24) have been observed in this period. Note that we can see here some hint of a periodicity of about 60–70 yrs.

Figure 8 (left) shows the time dependence of the relative fractions of sunspot groups of various sizes. The 11-year (132 month) cycle is most clearly manifest in the plot for the large sunspot groups, but is also fairly clear in the plot for the small groups. It is more difficult to distinguish this cycle in the groups of intermediate area. To more precisely answer the question of whether an 11-year periodicity is observed in the incidences of groups of various areas, we constructed periodograms separately for the incidences of groups of each area class, shown in the right-hand side of Fig. 8. It is often believed that large groups can be manifest at any phase of the solar cycle. Somewhat unexpectedly, our periodograms shown that the 11-year cycle is most clearly expressed for large groups. It is also clearly expressed for small groups. However, the groups of intermediate area

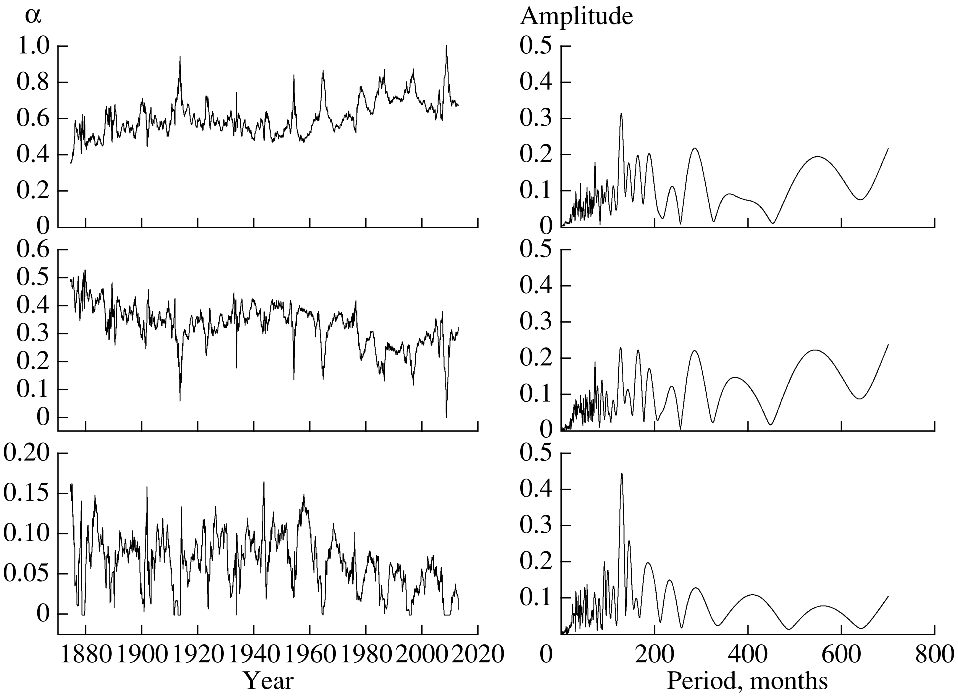

Fig. 8. Left: time dependence of the relative fractions of sunspots of various areas (small, medium, and large from top to bottom). Right: periodogram of the incidences of groups of various areas (same order as the right-hand plots).

do not show evidence for this periodicity in their incidences. Another curious fact is that 22-year and 60-year cycles are visible in the periodogram for small spot groups. This is probably a manifestaton of the fact that small groups are closely related to global magnetic-field generation processes.

4. CONCLUSION

We have analyzed data from the extended Greenwich Catalog in order to investigate the dependence of 11-year and longer-period variations in the numbers of sunspot groups on their areas. Small and large sunspot groups form two different populations. This is reflected in the different heights for their 11-year cycles, as well as for the longer-period variations. The contribution of large groups is higher at the beginning of the studied time interval (the end of the high 11th cycle) and during the anomalously high 18th and 19th cycles (between 1945 and 1960). During the periods of modest cycles in 1910–1930 and 1985–2000, the fraction of large sunspot groups fell (Figs. 4 and 5). Somewhat unexpectedly, the 11-year cycle is most clearly expressed for both large and small sunspot groups, but is not clearly distinguishable for the groups of intermediate area. Another curious fact is that the longer 22-year and 50-year cycles are visible in the data for small and medium sunspot groups, while they are not evident in the data for large groups (Fig. 8).

The fraction of small sunspot groups with areas to 100 msh possessing umbrae, α_1, has a maximum at the beginning of the 1930s, relative to the total number of sunspot groups. A minimum in the number of special groups with sunspots that do not possess umbrae, α_2, occurs at this same time.

Our results show that, despite the fact that sunspot groups on various scales are undoubtedly different manifestations of the same solar-activity phenomenon, their statistical properties are clearly different. Although the 11-year cycle is the main determining process for sunspot groups of various areas, the heights and heirarchy of the cycles observed for groups with large and small areas are different. The highest cycle for the large groups is Cycle 19, as for the Wolf number, while the 19th cycle is only average for the small groups, and the maximum cycles for these groups are Cycles 21 and 22.

The relationship between sunspot groups of various sizes is very important for the theory of the solar magnetic dynamo. Since it is known that sunspot groups pass through periods of growth and decay during their lifetime, a small group could represent either the beginning or the end stage in the evolution of the group. It seems likely that magnetic field that forms in the base of the convective zone rises into subsurface layers with a modest strength of several hundred Gauss. Subsequent amplification occurs in the subsurface layers, via processes that are not yet fully understood. Thus, the relationship between sunspot groups of various sizes could shed light on relationships between processes associated with the primay and secondary dynamos.

The regular behavior we have found supports the earlier assertion that the formation of groups does not end with the appearance of small spots on the solar surface. Additional amplification of a group occurs in surface layers, with the regularities of this transition probably being governed by a secular cycle.

Thus, variations in the relative fractions of sunspot groups of various sizes reflect a certain long-period variation in the spot-forming activity of the Sun. It was expected a priori that the contrbution of large sunspot groups should vary with the 11-year cycle. We have essentially found evidence for a secular mechanism that acts on small spot groups. It may be that the variations in the number of small spots characterizes the deep dynamo, while the formation of large spots is determined by processes in the subsurface layers. This result is of fundamental importance for the theory of the generation of solar activity.

Note also that the Maunder minimum was determined using indirect data or the numbers of sunspots accessible to low-resolution observations; i.e., these were most likely sunspots in large groups. This may mean that the deep dynamo continued to function in the Maunder minimum, while the subsurface dynamo was “switched off.”

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