AMPLITUDE AND PERIOD OF THE DYNAMO WAVE AND PREDICTION OF THE SOLAR CYCLE

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Abstract. The relation of the solar cycle period and its amplitude is a complex problem as there is no direct correlation between these two quantities. Nevertheless, the period of the cycle is of important influence to the Earth's climate, which has been noted by many authors. The present authors make an attempt to analyse the solar indices data taking into account recent developments of the asymptotic theory of the solar dynamo. The use of the WKB method enables us to estimate the amplitude and the period of the cycle versus dynamo wave parameters in the framework of the nonlinear development of the one-dimensional Parker migratory dynamo. These estimates link the period T and the amplitude a with dynamo number D and thickness of the generation layer of the solar convective zone h. As previous authors, we have not revealed any considerable correlation between the above quantities calculated in the usual way. However, we have found some similar dependences with good confidence using *running* cycle periods. We have noticed statistically significant dependences between the Wolf numbers and the *running* period of the magnetic cycle, as well as between maximum sunspot number and duration of the phase of growth of each sunspot cycle. The latter one supports asymptotic estimates of the nonlinear dynamo wave suggested earlier. These dependences may be useful for understanding the mechanism of the solar dynamo wave and prediction of the average maximum amplitude of solar cycles. Besides that, we have noted that the maximum amplitude of the cycle and the temporal derivative of the monthly Wolf numbers at the very beginning of the phase of growth of the cycle have high correlation coefficient of order 0.95. The link between Wolf number data and their derivative taken with a time shift enabled us to predict the dynamics of the sunspot activity. For the current cycle 23 this yields Wolf numbers of order 107 ± 7.

1. Introduction

The one-dimensional toy-model of Parker's (1955) migratory dynamo enables us to reproduce basic properties of the equatorward dynamo wave. This wave is well known to be indicated by sunspots over the solar cycles. Besides a series of comprehensive numerical studies (e.g., the review Brandenburg, 1994) there is an asymptotic solution for the linear Parker's dynamo in an inhomogeneous background of helicity and differential rotation recently found by Kuzanyan and Sokoloff (1995). While this model is generally in agreement with solar observations (Kuzanyan and Sokoloff, 1997), it cannot reproduce the internal structure of the solar cycle, i.e., the asymmetry between the phases of growth and decay of the sunspot indices.

However, the real nature is nonlinear, and for comparison with observations we might apply some results of nonlinear studies of such dynamos. There are compre-

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hensive numerical studies, e.g., Jennings and Weiss (1991), Tobias (1995), Tobias, Proctor, and Knobloch (1997), as well as asymptotic ones, e.g., Meunier *et al.* (1997) and Bassom, Kuzanyan, and Soward (1999). One should note that numerical results are often harder to compare with solar observations as they are usually influenced by a number of specific effects which the researchers wished particularly to investigate in a specific numerical model. Nonetheless the asymptotic methods give rough but general trends which can be checked by the observational data. Thus the asymptotics may reasonably complement numerical studies of mechanisms of the solar dynamo.

This paper intends to apply the results of such asymptotic studies of the nonlinear dynamo wave to reveal regularities in long-term series of the indices of solar activity and to compare the theoretical and observational trends. We study the relation of the length of the solar cycle and duration of its phases with amplitude of the cycle. This problem was repeatedly considered since long ago (see, e.g, Waldmeier, 1935; King-Hele, 1963, 1966). We are carrying out an attempt to approach this problem taking into account recent results on the asymptotic properties of dynamo waves cited above.

2. Basic Formulae

The estimates of the amplitude a and the period T of the $\alpha\Omega$-dynamo as obtained in asymptotic studies of Meunier *et al.* (1997) and Bassom, Kuzanyan, and Soward (1999) are as follows:

$$a \sim \sqrt{D - D_c} \quad \text{and} \quad T \sim (D_c)^{\kappa} \, , \qquad \text{(1a, b)}$$

where

$$D_c = d_c/\epsilon^3 \quad \text{and} \quad \epsilon = h/R \, ; \qquad (2)$$

d_c is a constant, ϵ is the aspect ratio of the generation layer of the convection shell, h its thickness, and R the internal radius of the convection shell.

We have denoted D as the dimensionless dynamo number, i.e., the regeneration rate of the magnetic field. This is a large in absolute value parameter for the solar dynamo (say, 10^3–10^4). D_c means the critical value of D when there is an onset of generation of a finite amplitude wave. The estimates (1a, b) presume a weakly-nonlinear case, of which the framework of validity were studied by, e.g., Bassom, Kuzanyan, and Soward (1999). We assume that for the Sun the dynamo number does not exceed the allowed limit of weak nonlinearity. Thus we assume that the magnetic field energy is relatively small compared to the total energy density. We consider here only the first order of asymptotic approximation of the solution in which we see that the period of the dynamo wave depends very weakly on the excess of the dynamo number over the critical value (e.g., Meunier *et al.*, 1997).

For the weakly nonlinear regime we may neglect high order dependence and fix the cycle period by the threshold case. Then Equation (2) is valid provided that for exponent κ we assume a constant value $-\frac{2}{3}$ as given by linear theory (e.g., Kuzanyan and Sokoloff, 1997). Estimate (2) for the critical dynamo number is in agreement with asymptotic studies of Kuzanyan and Sokoloff (1996) and also numerical studies of, e.g., Moss, Tuominen, and Brandenburg (1990). This means that the thinner the generation layer of the convective shell h is, the higher is the threshold dynamo number D_c. For the constant d_c the calculations of, e.g., Kuzanyan and Sokoloff (1996), and Meunier *et al.* (1997) (see also Bassom, Kuzanyan, and Soward, 1999) yield $\sqrt{2^{11} \times 3^{-3}} \approx 8.71$.

One can see that both the amplitude and period of the dynamo wave depend on *two* independent governing parameters, namely D_c and D. Therefore, there is no direct link between a and T. This explains the failure of numerous attempts (e.g., Vitinsky, Kopecký, and Kuklin, 1986) to reveal any relation between the observational data on the amplitude and the length of the Wolf number cycle. Although simultaneous analysis of both a and T versus time in a long time series may reveal the trend of dynamics of these two parameters and reconstruct the proportionality constants required in estimates (1a, b) and the governing parameters D_c and D as slowly (compared with the duration of the cycle) changing functions of time. However, in this paper we restrict ourselves to consideration of different regularities.

Here we study the time series of the monthly mean Wolf numbers versus time since mid-18th century till present smoothed over 12 months (see, e.g., *ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS*). We are interested in the relation between the duration of different phases of the solar cycle with its total amplitude. Considering the solar dynamo as a nonlinear quasiperiodic process it is natural to expect that the higher the amplitude is, the more asymmetry there is between the phases of the growth and decay. Besides that, the duration of the cycle itself should reflect such asymmetry as well.

The beginning of the cycle is associated with a dynamo wave front which was studied asymptotically by Bassom, Kuzanyan, and Soward (1999). This study reveals the following dependences for the maximum amplitude $a_{\max}$ and the duration of the phase of growth τ,

$$a_{\max} \sim \exp \frac{c_0}{\epsilon} \quad \text{and} \quad \tau \sim \epsilon \,, \tag{3a, b}$$

where $c_0 > 0$ is a constant. Notice that the dependence of the amplitude on ϵ given by formula (3a) is much more significant than in Equation (1a). Let us assume that for the solar convection zone and its overshoot layer the values of ϵ may vary by a factor 2. Then for dynamo numbers D of order of the critical value D_c, the variation of the amplitude given by formula (3a) exceeds the one given by Equation (1a) by factor 10^2–10^3. Below we check this dependence with the series of the solar indices data.

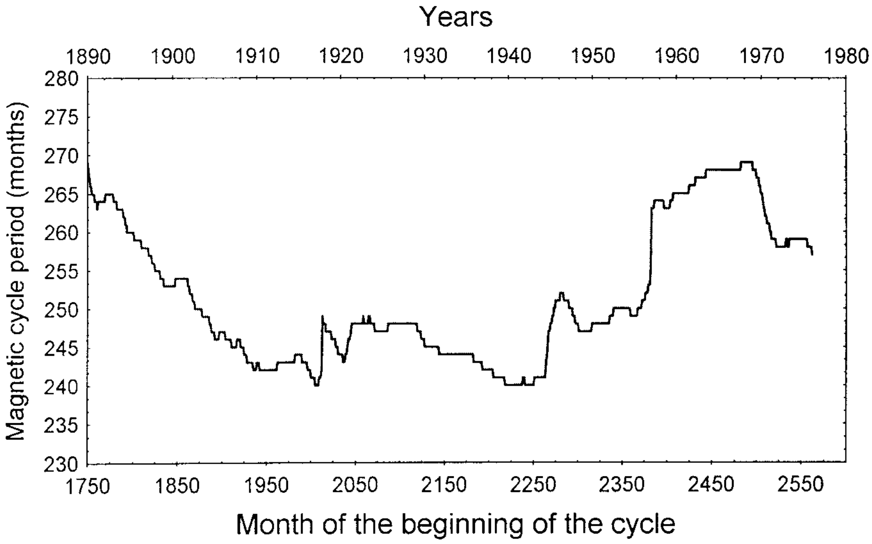

Figure 1. Running period of the magnetic Hale cycle calculated by the use of a cross-correlation technique. We used the limits of its length between 214 and 314 months.

3. The Analysis of the Data

In our analysis we used the data on the *running* period of the sunspot cycle calculated for every month from the series of the Wolf numbers by a search of the interval of maximum correlation. The *running* period of the cycle for month i was calculated by the following method. We calculated the autocorrelation function $ATF(i, \lambda)$ for an interval of length 60 values Rz_i $(Rz_i,\ Rz_{i+60})$ with a shift λ. We varied λ from 100 to 170 months. We determined the *running* period of the cycle as the value of λ which corresponds to the maximum ATF. The 'running' magnitude was calculated as an average value of Rz over months i to $i + \lambda$ using the formula

$$\overline{R}z_i = \frac{1}{\lambda} \sum_{k=i}^{i+\lambda} Rz_k\ . \tag{4}$$

A similar procedure is carried out for the magnetic 22-yr cycle where we considered the period of the Hale magnetic 22-yr cycle varied from 214 to 314 months. The resulting series of the *running* magnetic cycle periods is shown in Figure 1. We found that:

(1) The linear correlation of the *running* magnetic cycle period and the Wolf number is up to -0.68 for non-smoothed and -0.80 for smoothed time series of about three thousand numbers. The best correlations are attained for the magnetic cycle period with respect to Wolf number taken at time 134 months later in the period ($\approx$ 11 yr). The corresponding linear regression function is shown in Figure 2. Thus, the series of the cycle periods corresponds to the series of the Wolf numbers averaged over the 22-yr period shifted by approximately a half of this period.

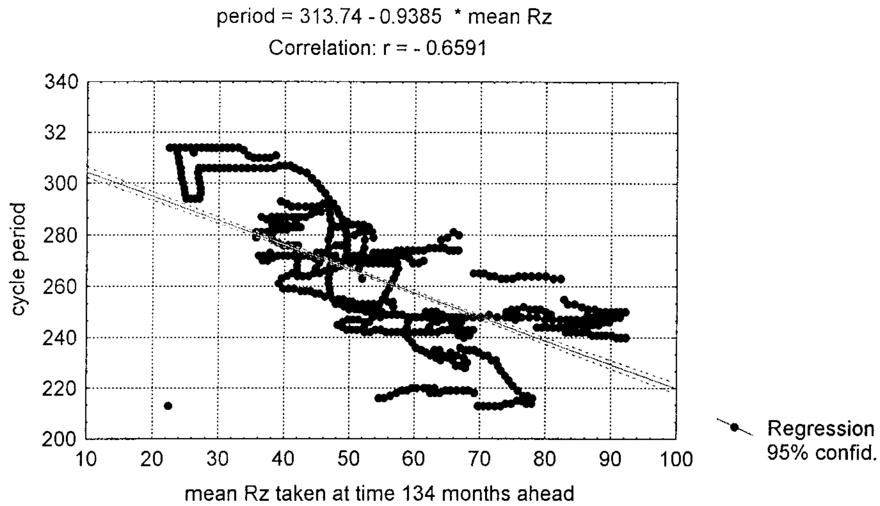

Figure 2. The linear regression function of the *running* Hale magnetic cycle period versus the mean Wolf number taken at time 134 months later.

A similar dependence has been found for the magnetic cycle period calculated by the usual ‘discrete’ way. The series of maximum Wolf numbers of the 22-yr magnetic cycle versus the cycle period taken with the corresponding shift is shown in Figure 3. One can see that the linear regression coefficient is -0.66 here. However, for the 11-yr sunspot cycle the correlation is lower, only -0.58 for the ‘discrete’ dataset, and even less for the *running* data series. Notice that without such a time shift high correlation cannot be attained.

(2) The analysis of asymmetry of the phases of growth and decay of the 11-yr sunspot cycle reveals good correlations of the duration of the ascending phase with maxima of Wolf numbers and maxima of the derivative of Wolf numbers calculated over a 12 month window ($\mathrm{d}Rz/\mathrm{d}t$). The linear correlation coefficients here are -0.73 and -0.71, correspondingly. However, the asymmetry of the phases has no remarkable correlation with the Wolf numbers. The duration of the descending phase does not show a significant correlation with any of the other parameters under investigation.

For the duration of the ascending phase of the cycle versus the maximum Wolf number we checked the dependences of formulae (3a, b). As one can see in Figure 4, the correlation (coefficient 0.77) is significant enough to support the asymptotic estimates obtained by Bassom, Kuzanyan, and Soward (1999). Thus we may conclude that the asymmetry of the phases of growth and decay of the cycle is mainly determined by the duration of the growth phase. The latter is shown to be linked with the maximum cycle amplitude.

(3) Besides that, we revealed regularities in the dynamics of the Wolf number derivatives with respect to time $\mathrm{d}Rz/\mathrm{d}t$ and the Wolf numbers themselves Rz.

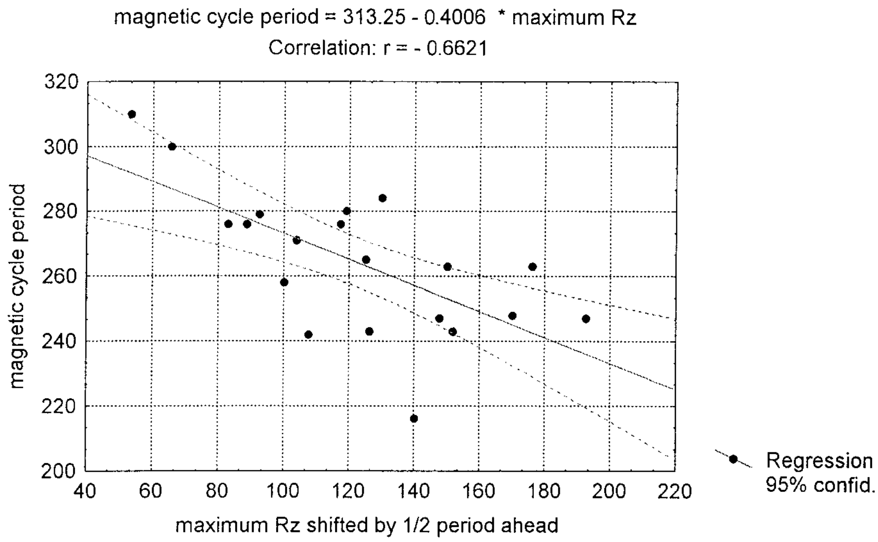

Figure 3. The same as in Figure 2 for the discrete series of magnetic cycle periods versus maximum Wolf numbers shifted by one-half period.

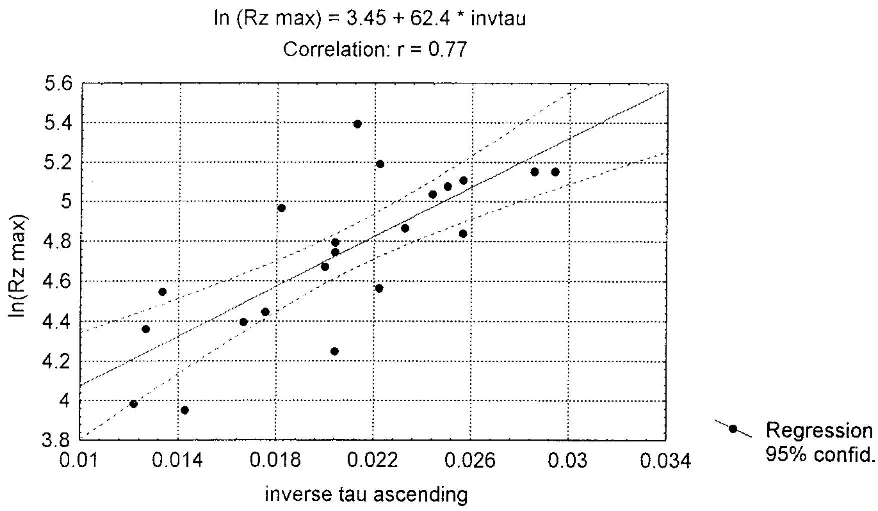

Figure 4. The linear regression function of the logarithm of the maximum Wolf number and the inverse ascending phase duration yields a correlation coefficient 0.77 supporting Equations (3a, b).

We found that the derivative dRz/dt is best correlated with Wolf numbers $Rz(t + 33)$ shifted ahead by an average of 33 months (see correlation coefficient versus time shift in Figure 5). The correlation coefficient for the monthly series over almost 250 years is surprisingly high (of order 0.7). This is in agreement with the nature of quasiperiodic oscillations, as the quantity is shifted with respect to its derivative by a quarter of a period (11 yr/4 $\approx$ 33 months). The linear regres-

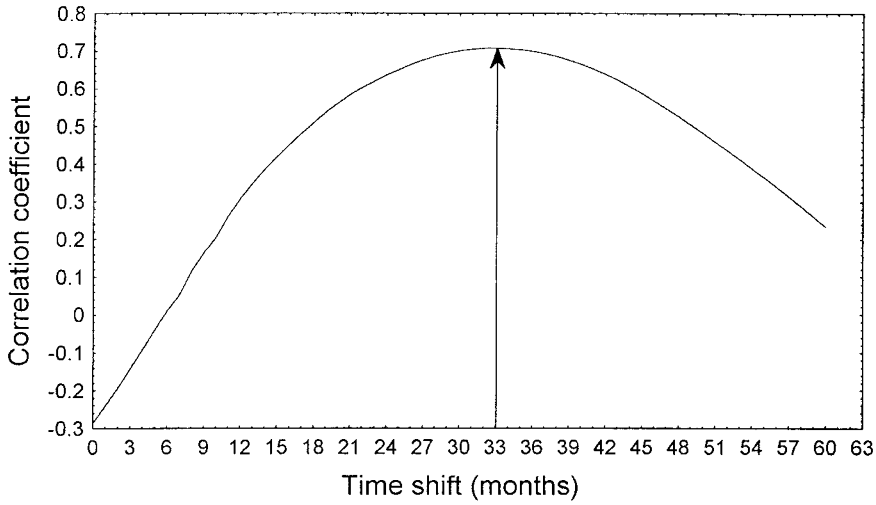

Figure 5. The cross-correlation function of series $dRz/dt(t)$ and $Rz(t)$ versus time shift.

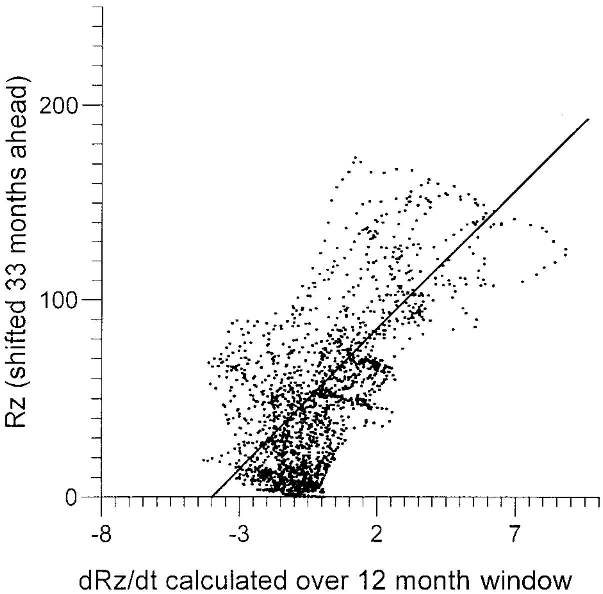

Figure 6. The linear regression function linking series $dRz/dt(t)$ and $Rz(t+\lambda)$ with a time shift $\lambda = 33$ months.

sion function linking series $dRz/dt(t)$ and $Rz(t)$ with this time shift is shown in Figure 6.

This means that we may try to predict the average amplitude of a given solar cycle by knowing the derivative of the Wolf numbers within previous periods. Another dependence of interest is represented in Figure 7. The maxima of the function of Wolf numbers over each given cycle were shown to have very good correlation

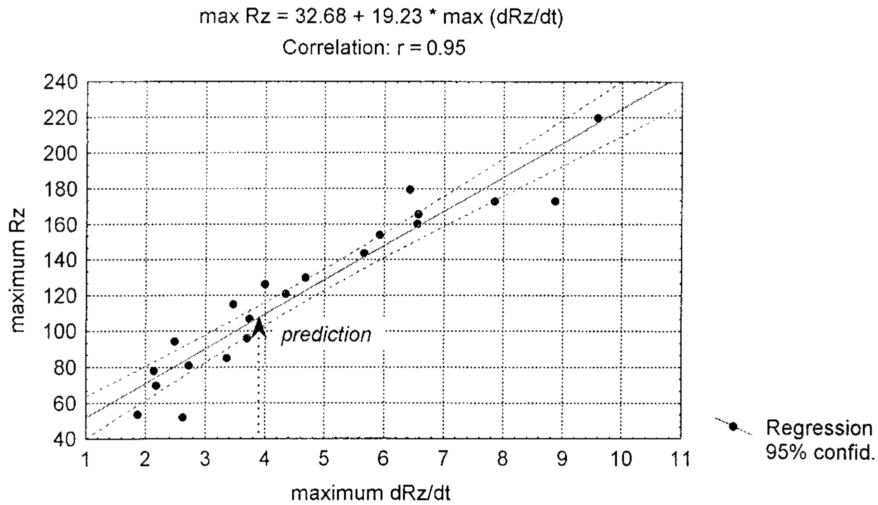

Figure 7. The linear regression function linking maxima of Wolf numbers with maxima of the 12-month window derivative for the same cycle. A prediction for cycle 23 is suggested.

TABLE I

Cycle maxima prognosis and forecast using the method.

Cycle number	Registered	Predicted
19	219.6	213 ± 20
20	120.7	114 ± 7
21	179.5	157 ± 10
22	172.9	180 ± 10
23	?	107 ± 7

(coefficient 0.95) with maxima of its derivative in the same cycle. These maxima of the derivative are located in the vicinity of the point $t_m A$ of the growing phase of the cycle as determined by Vitinsky, Kuklin, and Obridko (1986), Obridko and Shelting (1992).

Based on the high correlation between the maximum Wolf number over a given 11 yr cycle and maximum of its derivative calculated over a 12 month window, we predict the maximum Wolf number in the current solar cycle 23 to be of order 107 ± 7. The corresponding maximum derivative occurred in summer 1997, and on the basis of the best correlation time shift $\tau = 33$ months (see Figure 5) we may expect the maximum of cycle 23 to be most likely reached in spring 2000, probably in March–April. A comparison of the maximum Wolf numbers calculated for a few recent cycles by the use of this technique and registered ones is in Table I.

4. Discussion

We have noticed that Wolf number data are in good correlation with their temporal derivative taken with a time shift of on average 33 months. This enables one to predict dynamics of the solar sunspot cycle given the derivative of the smoothed sunspot numbers. The prediction for the maximum Wolf number for the current cycle 23 yields 107 ± 7.

Besides that we noticed that Wolf numbers significantly correlate with the duration of the solar cycle taken with a certain time shift. We also found a statistical relation between the duration of the phase of growth and maximum amplitude of the cycle. This supports understanding of the solar cycle in the framework of the nonlinear dynamo wave in the light of asymptotic studies of, e.g., Bassom, Kuzanyan, and Soward (1999), and also Meunier *et al.* (1997).

We have revealed a series of regularities in the temporal dynamics of the sunspot numbers. They enable us to consider the solar cycle as a deterministic quasi-periodic process. Such dependences may be understood as a message that the level of predictability of the solar dynamo mechanism is higher than it is usually considered. We might adopt the theoretical models in order to give a sufficient explanation to all such ‘strong’ dependences.

We can expect that the cycle period is controlled by the thickness of the generation layer (see formulae (1b), (2)). The shorter the period is, the thinner the layer. This hypothesis must be proved by comparison with observational data in forthcoming papers.

Acknowledgements

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