FOURIER PARAMETERS AND MOMENTS OF POLARIZATION PROFILES OF MAGNETICALLY ACTIVE LINES. FOURIER VECTOR MAGNETOGRAPH

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Abstract. A new method is proposed to determine all components of the solar magnetic fields using the cumulants of the profile of a magnetic sensitive line. The method is based on polarization measurements in a number of points of the line profile and subsequent calculation of the amplitudes and phases of its two first Fourier–harmonics.

Key words: Solar magnetic field – Fourier–magnetograph

1. Introduction

The methods employed to obtain vector magnetic fields on the Sun are usually based on measurements of the Stokes parameters in magnetoactive lines followed by interpretation with adequate radiative transfer modeling. As a matter of fact, one measures the polarized intensities either in selected regions of the profile (using Babcock–type magnetograph), or all over the profile (using Stokesmeter). The measurements can be carried out simultaneously or by scanning along the profile.

The magnetic field parameters, calculated from the measured polarization, are very sensitive to variations of the line profile due to the physical conditions in the line forming layer (such as temperature, pressure, and inhomogeneities), and strongly depend on the adopted model atmosphere.

The difficulties involved in the methods which employ a single wavelength can be avoided by using integral parameters of the profile (displacement of the gravity center, width, asymmetry etc.). Semel (1970) was one of the first to use relative displacements of the gravity centers of the σ–components of magnetoactive lines as a measure of the longitudinal magnetic field. The present work is, in some sense, an extension of this idea to the vector magnetic field.

Solar Physics **164**: 373–380, 1996.

2. Cumulants and their relations to magnetic fields

As integral parameters of the line we shall take the cumulants determined from the following relations:

$$æ_n = j^{-n} \left[\frac{d^n ln\tilde{I}(w)}{dw^n} \right]_{w=0}, \tag{1}$$

where $\tilde{I}(w) = \int I(\lambda) e^{jw\lambda} d\lambda$ is the Fourier transform of the line depression profile $I(\lambda)$, and $j = \sqrt{-1}$. There exist relationships between the cumulants and the central and starting moments of I (Malakhov, 1978):

$$\begin{aligned} æ_1 &= \alpha_1 && \text{- center of gravity of the line profile} \\ æ_2 &= \mu_2 = \alpha_2 - \alpha_1^2 && \text{- square of the width of the line profile} \\ æ_3 &= \mu_3 = \alpha_3 - 3\alpha_1\alpha_2 + 2\alpha_1^3 && \text{- asymmetry of the line profile} \end{aligned}$$

where

$$\mu_n = \frac{\int I(\lambda)\ (\lambda - \alpha_1)^n\ d\lambda}{\int I(\lambda) d(\lambda)},$$

are the central moments,

$$\alpha_n = \frac{\int I(\lambda)\ \lambda^n\ d\lambda}{\int I(\lambda) d(\lambda)},$$

are the starting moments, and $I(\lambda)$ is the line depression profile as a function of the wavelength measured from the nominal value. Though the first cumulants coincide with the central moments, we shall rather use the system of cumulants owing to their additivity in the convolution procedure (Malakhov, 1978).

As shown below, the knowledge of the two first cumulants of different states of polarization is sufficient to calculate all components of the vector magnetic field. Let us discuss one of the possible procedures. Let solar emission in a magnetoactive line with Stokes parameters I_0,V_0,Q_0 and U_0 be analyzed by a polarization analyzer consisting of 2 elements: a controlled birefringent plate and a linear polarizer. The operation of the analyzer will be described in the reference frame in which the axes coincide with those of the polarization ellipse, consequently $U_0 = 0$ *. The angle between the transverse magnetic field component and the plate axis is denoted by β, and the angle between the beam and the magnetic field vector by γ.

* In general, the polarization ellipse does not maintain a single orientation along the line profile and, consequently, such a single reference system cannot be found. However, some solutions of the radiative transfer equation do allow such a definition, in particular, those used at the end of this section.

The polarization analyzer has six states:

1) G $(\frac{\lambda}{4}, \beta)$; P $(\beta + 45^o)$ – delay between the ordinary and extraordinary beams produced by the plate is $\frac{\lambda}{4}$, angle between polarizer axis and the fast axis of the plate is 45^o. The arguments of G and P describe state of the retarder and the polarizer, respectively.

2) G $(-\frac{\lambda}{4}, \beta)$; P $(\beta + 45^o)$ – delay is $-\frac{\lambda}{4}$, angles are the same.

3) G $(0, \beta)$; P $(\beta + 45^o)$ – delay is zero, angles are the same.

4) G $(0, \beta)$; P (β) – delay is zero, polarizer axis turned by -45^o.

5) G $(0, \beta)$; P $(\beta - 45^o)$ – delay is zero, polarizer axis turned by -90^o.

6) G $(0, \beta)$; P $(\beta - 90^o)$ – delay is zero, polarizer axis turned by -135^o.

It is easy to show that radiation intensities at the analyzer output for each of its 6 states can be described as follows:

$$
\begin{aligned}
I^{(1)} &= 0.5 I_o - 0.5 V_o \\
I^{(2)} &= 0.5 I_o + 0.5 V_o \\
I^{(3)} &= 0.5 I_o - 0.5 \sin(2\beta) Q_o \\
I^{(4)} &= 0.5 I_o + 0.5 \cos(2\beta) Q_o \\
I^{(5)} &= 0.5 I_o + 0.5 \sin(2\beta) Q_o \\
I^{(6)} &= 0.5 I_o - 0.5 \cos(2\beta) Q_o .
\end{aligned}
\tag{2}
$$

Now, let us calculate the relationships between the original Stokes parameters and the cumulants for six states of the analyzer. For the sake of simplicity, assume that I_0 and Q_0 are strictly even functions of λ and V_0 is strictly odd function of λ. By substituting $I^{(1)}$ — $I^{(6)}$ in the expressions for moments we obtain:

$$
\begin{aligned}
æ_1^{(1)} &= -æ_1^{(2)} = -A(H, \gamma) \cdot \cos\gamma \\
æ_1^{(3)} &= æ_1^{(4)} = æ_1^{(5)} = æ_1^{(6)} = 0,
\end{aligned}
\tag{3a}
$$

$$
\begin{aligned}
æ_2^{(1)} &= æ_2^{(2)} = B(H,\gamma) - A^2(H,\gamma)\cdot\cos^2\gamma \\
æ_2^{(3)} &= \frac{B - C\sin 2\beta}{1 - D\sin 2\beta} \\
æ_2^{(4)} &= \frac{B + C\cos 2\beta}{1 + D\cos 2\beta} \\
æ_2^{(5)} &= \frac{B + C\sin 2\beta}{1 + D\sin 2\beta} \\
æ_2^{(6)} &= \frac{B - C\cos 2\beta}{1 - D\cos 2\beta},
\end{aligned}
\tag{3b}
$$

where

$$A(H,\gamma) = \frac{\int V_0 \lambda d\lambda}{\int I_0 d\lambda}\cdot\frac{1}{\cos\gamma}, \tag{4}$$

$$B(H,\gamma) = \frac{\int I_0 \lambda^2 d\lambda}{\int I_0 d\lambda}, \tag{5}$$

$$C(H,\gamma) = \frac{\int Q_0 \lambda^2 d\lambda}{\int I_0 d\lambda}, \tag{6}$$

$$D(H,\gamma) = \frac{\int Q_0 d\lambda}{\int I_0 d\lambda}, \tag{7}$$

and H is the magnetic field strength. However, the signals under consideration depend on properties of the atmosphere (pressure, temperature, velocity and magnetic fields, and the variation of these parameters along the line-of-sight). In order to simplify the computation, we adopt the same approximation as Unno (1956). Under this approximation, A, B, C, and D, depend on constant values of H (the field strength), γ (the inclination angle), η_0 (the line-to-continuum opacity ratio), g (the Lande factor), $\Delta\lambda_D$ (the Doppler halfwidth of the line) and the two coefficients which describe the source function. Figure 1 shows calculations based on this approach. The parameters have been chosen to correspond to those of the photospheric Fe I 5250 Å line. The ratio η_0 of opacities at the line center to that in the continuum was taken equal to 10. The field strength is given in units of the Doppler halfwidth ($H = 1$ corresponds approximately to 1000 G). A is expressed in Doppler halfwidths, B and C in square of the Doppler halfwidths.

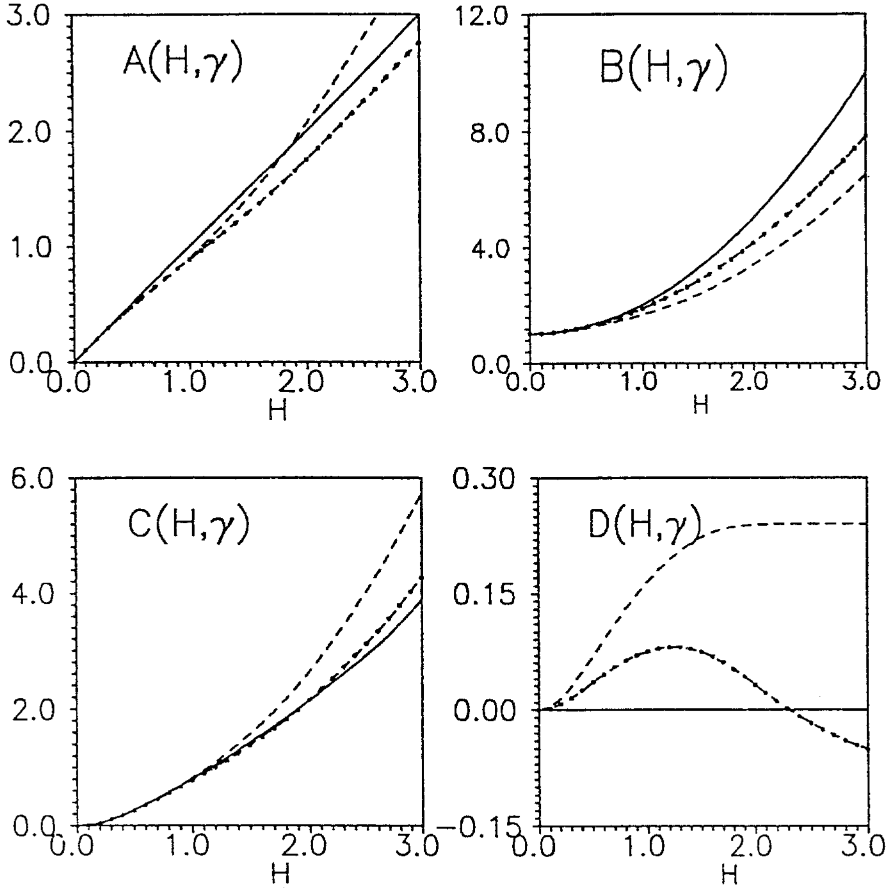

Fig. 1. Dependence of A, B, C, and D on the field strength H. The simulations refer to the line FeI 5250 Å observed in the photosphere, so that $H = 1$ corresponds approximately to 1000 G. The solid line is used for $\gamma = 0$, the dashed line for $\gamma = \pi/2$, and the dash–dotted line for $\gamma = \pi/4$.

3. Calculating cumulants by Fourier–analysis. Fourier vector magnetograph

Below we describe the method for calculating the cumulants by Fourier–analysis of the line profile. For this purpose, we use expressions connecting the amplitudes and phases of the Fourier harmonics of the line profile with its cumulants (Didkovsky, Kozhevatov and Stepanyan, 1986):

$$\Psi(\Delta\lambda) = \text{æ}_1 \frac{1}{\Delta\lambda} - \text{æ}_3 \left(\frac{1}{\Delta\lambda}\right)^3 \frac{1}{3!} + ...,$$

$$ln \frac{\tilde{A}(\Delta\lambda)}{S} = -\text{æ}_2 \left(\frac{1}{\Delta\lambda}\right)^2 \frac{1}{2!} + \text{æ}_4 \left(\frac{1}{\Delta\lambda}\right)^4 \frac{1}{4!} + ..., \tag{8}$$

$$S = \int I(\lambda) d\lambda,$$

where $\Delta\lambda = 2\pi/\omega$ is the period of the Fourier harmonic, and $\tilde{A}$ and Ψ are its amplitude and phase, respectively.

The two first cumulants are enough to determine the magnetic field under our simplifying assumptions (see Section 2). However, the determination of the first three cumulants might allow a more elaborate modeling (for instance, taking into account the asymmetry of the line profiles). Because of this future possibility, and since the determination of two or three harmonics requires a similar computational effort, here we describe how to compute $\neq_1$, $\neq_2$ and $\neq_3$. To determine the three first cumulants, we need to know the amplitudes and phases of the two first harmonics of the Fourier–transform of the line profile (the domain of definition of the line profile is considered as the period of the first harmonic). As seen from Equation (8), the phases of the long–period harmonics (those with small $1/\Delta\lambda$) are proportional to the shift of the center of gravity (æ_1). In order to determine æ_1 and æ_3, we have to know the phases of two harmonics (with small $1/\Delta\lambda_1$ and with somewhat larger $1/\Delta\lambda_2$). A similar procedure is applied to determine the even cumulants from the amplitudes. The period of the first harmonic is large enough, so that the first term dominates the series for the phases. The term $\Delta\lambda^{-3}$ and higher order terms can be neglected, and consequently, $\neq_1$ is determined from the phase of the first harmonic. The second harmonic has a smaller period; in this case the series for the phases has two major terms while the series for the amplitudes has just one term. Since the first cumulant and S are known, the expressions for the phase and amplitude of the second harmonic are used to derive the third and the second cumulants.

According to the Kotelnikov – Shannon criterion (see Bell, 1972), the characteristics of the Fourier harmonics can be determined by making records at discrete points spaced by no more than 1/3 of the harmonic period. Therefore, the intensities measured at six points along the profile are enough to determine the amplitude and phase of the two first harmonics. In this case one condition should be observed: before discretization, higher–frequency harmonics must be eliminated from the line profile to avoid aliasing (see, e.g., Bracewell, 1978). The smoothing can be done by selecting the appropriate resolution of the instrument.

The procedure of determining magnetic fields by the proposed Fourier–method comprises the following stages:

1. Measure intensities at six discrete equidistant points along the line profile at six positions of the polarization analyzer.

2. Calculate the phases and amplitudes of the two first harmonics of the line profile.

3. Use these to find two or three first cumulants of the line profile.

4. Determine the magnetic field parameters from Equations (3)-(7). These expressions yield at least 6 independent equations for 3 unknown values, H, γ, and β. Consequently, one expects no unique solution and, therefore, some kind of least squares best fit has to be sought. This procedure has not been developed yet.

Note that parameters of the Fourier–harmonics can be obtained by a different method than proposed in items 1 and 2 above, for example, directly by using Michelson interferometers (Bell, 1972; Didkovsky, Kozhevatov and Stepanyan, 1986).

4. Conclusions

We believe that the proposed method for determining magnetic field parameters from the cumulants of the line profile offers a number of advantages, compared with the traditional Babcock method, based on the comparison of polarized light intensities at two symmetric points in the line profile.

1. The relation between the cumulants and the magnetic field value does not show saturation or back pass at large field intensities.

2. The magnetic field is determined from integral properties of the line profile, so that the result will not be affected strongly, by the asymmetries of the profile.

3. The use of integral properties will considerably reduce the effect of spatial irregularities (see Rees and Semel, 1979).

4. When determining the magnetic field with a Babcock–type filter magnetograph, the presence of non–compensated line-of-sight velocity may result in significant errors. Our method, in which the effects of the magnetic field and the line–of–sight velocity are separated, does not have this disadvantage, and to a certain extent, it resembles the photographic method. (For detailed description of the photographic method see, for example, Bray and Loughead, 1964)

5. Our method requires measurements at a limited number of points along the line profile.

Acknowledgements

We thank Jorge Sánchez Almeida for valuable comments on the manuscript.

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